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Economics of Inventory Control

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Inventory control is important to management in almost all enterprises, including Government operations which are financed with public funds. This article discusses mathematical and quantitative methods of helping managers in this important function.

The goals of inventory management are to:

—provide stock to users when and where it is needed,

—keep investment in inventory low, and

—obtain the lowest available prices for items purchased, quality considered.

The second goal conflicts with the first and the third. Buying in large quantities reduces the probability of stockouts, reduces the cost of placing and receiving replenishment orders, and tends to result in lower prices and transportation costs. But the larger quantities tend to increase investment and the cost of holding the inventory.

The inventory manager must seek a balance between the costs of holding inventory, ordering additional quantities, and running out of stock in deciding:

—when to acquire additional stock and

—how much to acquire.

Each of these decisions involves risks. Both require forecasts of future demand and leadtime to acquire additional stock. Such forecasts are seldom free of error. Demand usually varies from month to month and delivery time from order to order. If he is to achieve a reasonable balance between the costs, the manager must assess the probability of an error in his forecasting and provide for it.

The Structure of Inventory Systems

Knowledge of the properties of a

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system is essential for obtaining an inventory policy which will adequately represent the system. Omitting properties, or making wrong assumptions about properties, will usually lead to improper control of inventory. In establishing policy, the inventory manager should consider at least the following properties.

1. *The inventory's spatial configuration*: For example, do demands occur at a single outlet or at a number of outlets? Does the system have one, two, or several sources of supply?
2. *Resource constraints affecting inventory*: For any inventory system, the manager must know the nature of budgetary restrictions, limitation of available warehouse floorspace, or any other relevant forces.
3. *The kinds of items carried*: Some inventory systems handle only a single item while others carry as many as 1,500,000. These items may be reparable or nonreparable; they may become obsolete very rapidly or they may be staple items.
4. *The processes generating requirements*: The rate of demands for inventory items can rarely be predicted with certainty. How well an inventory system operates depends to a considerable extent on how much its managers learn about the demand generation process.

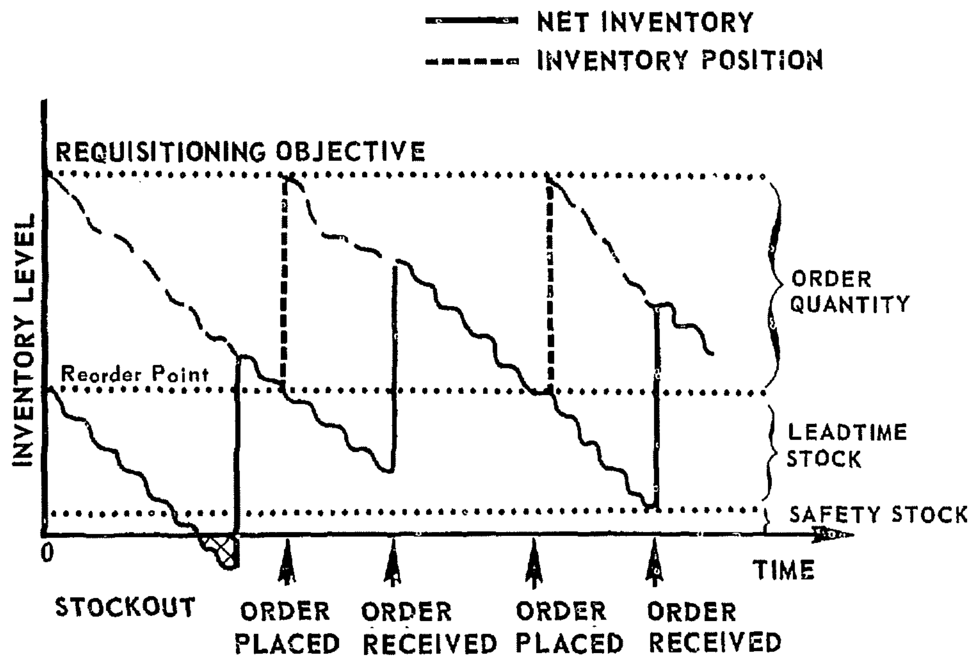
Sophisticated techniques and mathematical models have been developed for solving inventory control problems. These techniques are described extensively in a number of publications. One that is very readable is "Production and Inventory Control Principles and Techniques" by G. W. Plossl and O. W. Wight, Prentice Hall, 1967.[1] For inventories containing very large numbers of items, full employment of these techniques, many of which are beyond the scope of this paper, requires a computer to handle the thousands of computations involved. But some application of the techniques are practicable without a computer—at least for the relatively few items used in large quantities. Any system for controlling inventory should be closely monitored so that it can be adjusted as necessary to obtain the desired results.

When To Buy

Obviously, if the manager waits to replenish stock until all on hand is consumed, he will be out of stock while a replenishment order is being placed, processed by the supplier, delivered, inspected, and placed in stock. To avoid such stockouts he must estimate how much stock will be needed so that a replenishment order can be placed when stock on hand reaches a certain point. Since both use and time to process replenishment orders may vary from the estimate, the reorder point should ordinarily be increased to include a quantity for safety.

The inventory use and replenish-

[1] See also: "Basic Principles of Wholesale IMPACT-Inventory Management Program and Control Techniques," 2d ed., IBM, March 1972.



ment cycle is presented in the following graph.

The inventory manager must set reorder points with care because, if they are too high, holding costs are excessive and, if they are too low, the costs of stockouts are excessive. In trying for a balance between these two consequences, the manager must answer a number of interrelated questions and put them together in a rational way.

A straightforward estimate of the quantity to be consumed during leadtime can be made after two questions are answered.

1. How long does it take to replenish the stock? This is an estimate based on experience with the item concerning the time required to place an order, have it processed and delivered by the supplier, inspect it, and place it in stock ready for issue.
2. What is the rate of consumption? Such an estimate is frequently based on the average usage during prior periods.[2]

Setting safety levels, the remainder of the reorder point stock, requires answers to three additional questions.

1. How often can stockouts be tolerated? This may be expressed as number of stockouts a year. In theory, the answer should be designed to balance the cost of being out of stock[3] and the

[2] Demand forecasts are frequently made using smoothing constants. Under a method commonly referred to as exponential smoothing, a forecast for a future period is comprised of X percent of consumption during the most recent period plus 1-X percent of the previous forecast. Setting X at 15 percent simulates about a 12 period moving average. Smoothing constants are generally considered equal to $2/(1+\underline{n})$ where $\underline{n}$ is the number of periods to be simulated.

[3] Examples of the kinds of costs involved will be given later.

cost of holding the safety stock. As a practical matter, the cost of being out of stock can seldom be determined. Therefore, the number of tolerable stockouts is usually a policy decision subject to adjustment as experience is gained on an item.

2. How often is the stock to be replenished? This is the quotient of estimated annual demand divided by the order quantity. As shown by the preceding graph, a stockout can occur during any replenishment cycle. Frequent replenishments increase the risk of stockouts.

3. How reliable are the estimates of periodic demand and leadtime? This requires an estimate of the standard deviation of periodic demands, a statistical measure of the periodic variations in demand. For the mathematically inclined the standard deviation is equal to

$$\sqrt{\frac{\Sigma(X-\overline{X})^2}{N-1}} \quad \text{where}$$

Σ = summation

X = demand for each period

$\overline{X}$ = average demand

N = number of periods considered

Forecasts of demand can be substituted for average demand in the formula. Another simpler method is to estimate the standard deviation as equal to 1¼ times the average deviation from the periodic forecasts. For this purpose the differences between actual and forecast demand are added regardless of whether the forecasts are over or under actual demand.

The answers to questions 1 and 2 are necessary to determine the desired probability established by management that a stockout will not occur during a replenishment cycle. For example, if management establishes an objective of one stockout a year and stock is to be replenished 20 times a year, the probability of a stockout must be reduced to 1/20 or 5 percent. Stated another way, the probability that stock will not be exhausted must be 95 percent.[4]

The answer to when to buy is: at the time the stock on hand plus any undelivered quantity on order reaches the reorder point. The reorder point should include leadtime stock and adequate but not excessive safety level stock.

How Much To Buy

This decision requires consideration of at least:

—estimated periodic demand,
—cost to process an order,
—cost to hold the inventory, and
—quantity discounts.

If quantity discounts are not available, that is if the price for the item including transportation cost is not affected by the size of the order, the objective is to minimize annual ordering and holding costs. These costs are lowest when they are equal. To deter-

[4] This percent is important because it is a guide to the factor (1.65 for 95 percent) which multiplied by the estimated standard deviation during leadtime should show the quantity needed for safety stock.

mine the order size which will equalize these costs, the manager must solve the economic order quantity (EOQ) formula.

The basic formula is relatively simple and can be easily incorporated into a computer program:

$$Q = \sqrt{\frac{2DP}{IC}} \quad \text{where}$$

Q = the economic order quantity
D = annual demand
P = the cost of processing an order
C = price of the item
I = holding cost per year as a percent of average inventory investment

The formula has a number of expansions to fit different circumstances. For example, if two or more items are to be acquired in a single order, the formula becomes

$$Q_i = D_i \sqrt{\frac{2P}{I(\Sigma D_i C_i)}} \quad \text{where}$$

Q_i = the economic order quantity for any of the items
D_i = quantity of annual demand for any of the items
P = cost of processing the order
$(\Sigma D_i C_i)$ = value of annual demands for all the items combined in the order

This expansion recognizes that under a combined order the ordering cost for each item is reduced. For example, consider the following assumed data:

	Items		
	A	*B*	*Combined*
Ordering cost	$ 25	$ 25	$ 40.50
Holding cost	15%	15%	15%
Annual demand	$59,820	$22,320	$82,140.00
EOQ (combined orders)	$ 4,845	$ 1,806	$ 6,651.00
EOQ (separate orders)	$ 4,465	$ 2,725	$ 7,190.00
Ordering and holding cost:			
combined orders			$ 1,000.00
separate orders	$ 770	$ 409	$ 1,179.00

If a computer is not available, tables can be constructed for ready reference using a desk calculator. A convenient form relates demand to months of supply as indicated below.

Annual demand	*Months of supply*
$ 5,001— 10,000	6
$10,001— 15,000	4
.	.
.	.
.	.
$50,001— 60,000	1

Quantity Discounts

Suppliers frequently offer lower prices if orders exceed stipulated quantities. If such discounts are available, they may offer greater economy than the balancing of ordering and holding costs. Their availability complicates the process of determining the optimal purchase quantity, but not unduly. The process may require one or a series of trial computations.

Step 1. Use the EOQ formula and

calculate a quantity at the lowest price.

a. If the lowest price is available at that quantity, it is the optimal purchase quantity.

b. If the quantity is less than the price break quantity, compute the annual cost of buying the price break quantity and proceed to step 2.

Step 2. Use the EOQ formula and calculate a quantity at the next lowest price.

a. If the price is available at that quantity compute the annual cost of buying that quantity and compare this amount with the amount computed in 1(b). The lower of the two shows the optimal quantity.

b. If the quantity is less than the price break quantity, compute the annual cost of buying the price break quantity. Keep repeating the step 2 process, comparing annual costs until the price used in the EOQ formula is available for the computed quantity. The optimal quantity is that which results in the lowest annual cost. It will always be equal to or greater than the EOQ quantity.

Example: The following data applies to a low-cost item.

Annual demand	600
Ordering cost	$8.00
Holding cost	20%
Prices available for orders of:	
less than 500	$0.30
500 but less than 1,000	0.29
1,000 or more	0.28

The results of the process described above are shown in the following table.

	Step 1	*Step 2*	*Repeated step 2*
Price break quantity	1,000	500	Less than 500
Price	$ 0.28	$ 0.29	$ 0.30
EOQ	414	406	400
Price available at EOQ	No	No	Yes
Annual cost:			
Price available at EOQ	$168.00	$174.00	$180.00
Ordering cost	4.80	9.60	12.00
Holding cost	28.00	14.50	12.00
Total	$200.80	$198.10	$204.00

In this case, the expected annual cost of buying 500 is less than that of buying the EOQ amount and the optimal quantity occurs at a price break. But the 28 cent price is not the bargain it might appear because increased holding costs for buying 1,000 more than offsets the 1 cent price savings.

Whether to make the additional investment to obtain a price break naturally involves consideration of such constraints as limited storage space, shelf life, and limited funds. If a number of items are considered together and funds are not available for all price breaks, a logical choice is to select those that offer the greatest return on the additional investment.

Determining What Costs To Use

Holding, ordering, and stockout costs should be included to the extent that they can be expected to vary by inventory investment, orders placed, and stockouts. As a practical matter, the inventory manager will usually have to rely on the best estimates of such costs that can be developed from available cost accounting records.

Holding cost is the cost of physically storing the inventory plus the cost of the capital required for inventory investment. It is customary to compute holding costs as a fraction of the cost of items carried in inventory per unit of time. This estimate of cost should include all costs that vary with inventory size, such as:

1. The cost of storage, including allowances for deterioration, obsolescence, and maintenance while in storage.
2. The cost of money tied up in inventories.
3. The cost of taxes on inventories.
4. The cost of insurance of inventories.

Ordering cost (sometimes referred to as setup or replacement cost) is the variable cost of placing an order (or setting up production). The formula implies that it is independent of the size of the order, but higher costs should be used in the formula for items requiring additional procurement or quality control. Ordering generally includes:

1. Requirements review.
2. Contract placement and administration, including inspection and acceptance.
3. Preparation, recording, and submission of requirements requisitions and purchase requests.
4. Control and documentation efforts.
5. Receiving shipments.

Shortage cost is the cost associated with being out of stock when an item is demanded. This cost may be impossible to determine because it involves a number of intangibles. The inventory manager may be forced to rely on intuitive judgments to decide how frequently stockouts can be tolerated without determining their cost. If this cost is estimated some of the costs that should be included are

1. Overtime costs.
2. Special clerical and administrative costs.
3. Loss of specific sales.
4. Loss of good will.
5. Loss of customers.
6. Equipment out of commission for lack of parts.

Effect of Errors in Estimating Costs

An examination of the EOQ formula will show that the optimal order quantity is not very sensitive to changes in any one of the variables. The EOQ increases less than proportionately with increases in the ordering cost P and demand rate D, and decreases in the holding cost I. For example, if the rate D quadruples, and the other variables remain the same, then the EOQ only doubles. Analogously, if the value of I quadruples, then the EOQ only decreases by a half.

Although ordering and holding costs should be as accurate as practicable, consistent with a certain amount of averaging, the small errors inherent in the process do not invalidate its use. A rule of thumb followed by some managers is that, if an error in these factors does not make more than 10 percent change in the expression $\sqrt{\frac{2P}{IC}}$[6], the error is insignificant.

In many situations the vendor requires that the amount ordered be a "convenient" number. Because the EOQ formula is not very sensitive to changes in cost estimates, there is very little impact on total cost when only a near optimal quantity is acquired. For example, if the optimal quantity is 53, an order of 50 or 60 may be a more desirable amount.

Summary

Effective inventory management requires an assessment of probabilities and costs or consequences. Although some judgments may have to be made more or less intuitively, management can usually be refined after some experience with the items.

Armed with adequate information on historical demand, ordering and holding costs, quantity discounts, and appropriate techniques of analysis, the inventory manager is in a position to maintain the inventory at levels which keep risks to tolerable limits but do not inordinately increase costs.

If the manager is faced with such constraints as limited ability to place orders, limited storage space, or limited procurement funds, sophisticated techniques of aggregate inventory management are available which permit optimal solutions within the constraints. These usually require use of a computer to handle the thousands of computations they employ.

Some constraints are usually necessary in any system to limit (1) procurements of shelf-life items or years of supply of slow moving items and (2) reorder frequence.

Use of the techniques described cannot be expected to result in a foolproof system. They have in many instances, however, provided management increased ability to establish and monitor inventory policy.

[6] Same as the EOQ formula but excluding annual demand.