

Use Of Statistical  
Sampling Techniques

*UNITED STATES*  
*GENERAL ACCOUNTING OFFICE*

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Sampling Techniques**

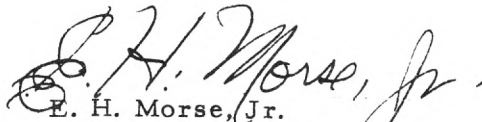
UNITED STATES  
GENERAL ACCOUNTING OFFICE  
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## FOREWORD

Statistical sampling techniques, properly applied, can contribute significantly to obtaining reliable analyses in our audit work with a reduction in manpower. These techniques also enable us to project our findings with precision and confidence.

This pamphlet describes a sampling plan used in an actual GAO audit and describes some alternative plans that could also have been used with additional savings in manpower.

This material is being distributed to highlight the usefulness of these techniques and to encourage wider use of them, where appropriate, in our audit work.

  
E. H. Morse, Jr.  
Director, Office of Policy  
and Special Studies

September 15, 1968

Basic distribution in GAO:

GS-11's and above in the accounting  
and auditing divisions

Transportation Division - 10

## ACTUAL AUDIT PLAN AND RESULTS

In an audit at 12 agency locations, statistical sampling techniques were used to estimate the "error" rates at each of the locations.

The audit program specified the need for reliable error rate findings at each of the locations visited. In order to satisfy this objective, a fairly large sample size was needed at each location.

Attribute "Sample Size" tables were used to obtain sample sizes. The criteria that were used to determine the sample sizes needed were: (\*)

1. Expected error rate not worse than: 10%
2. Desired sampling precision:  $\pm$  (plus or minus) 3%
3. Desired confidence in sample results: 95%

(\*) See Table 2-A, page 7-13, Audit Sampling Memorandums

The above criteria were uniformly applied in arriving at sample sizes for 11 of the selected locations. A 100 percent audit was specified for one location with a universe of 190.

The following table summarizes the audit sampling plan and actual error rate findings:

T A B L E A

Location	Number in Universe	Number in Sample	Sample Items With Error		Sampling Precision at 95% Confidence	Estimated Total Items With Error
			Number	Percent		
1	4,183	370	44	11.9%	±3.2%	500
2	4,498	356	56	15.7	3.6	710
3	2,350	318	15	4.7	2.2	110
4	19,721	377	127	33.7	4.7	6,650
5	14,145	375	123	32.8	4.7	4,640
6	3,525	354	172	48.6	4.9	1,710
7	14,705	372	136	36.6	4.8	5,380
8	5,148	328	27	8.2	2.9	420
9	4,366	308	31	10.1	3.2	440
10	2,810	355	5	1.4	1.1	40
11	913	353	71	20.1	3.3	180
12 <sup>a</sup>	190	190	33	17.4	-	33
	<u>76,554</u>	<u>4,056</u>	<u>840</u>	<u>27.2%</u>	<u>±1.8%</u>	<u>20,813</u>

<sup>a</sup>No sampling variability because of 100 percent audit.

Note: Individual location error rates were obtained by dividing the number of errors found at the location by the total number of items in the location sample. The overall error rate was obtained by dividing the estimated total items with error by the number of items in the universe.

It may be recognized from the above table that the overall sampling precision is not a simple average of the sampling precision obtained at each of the locations. Sampling precision is obtained by use of the standard error formula for stratified sampling (see Audit Sampling Memo 9-9). The disproportionate assignment of the total sample among individual locations, in effect, establishes 12 separate strata for sampling purposes. The calculation results in an overall error of 1.8% at the 95% level of confidence. Thus, in this case, it may be stated with 95% confidence that the number of errors in the universe is from 25.4% (27.2 - 1.8) to 29.0% (27.2 + 1.8) of the universe or from 19,445 to 22,201.

While the agency has several hundred individual locations, it was not intended that our audit findings be projected

agency-wide. The final report presents our audit findings for the 12 locations and contains selected individual location findings as examples of the various types of errors found. The appendix to the report includes a detailed summary of individual location findings.

The audit performed is a common repetitive type and is well suited for a presentation which shows how various sample plans could have effectively reduced the amount of the overall time required to perform the audit.

The alternate plans presented below are affected primarily by the audit objectives. The alternate plans are practical and worth considering in future audit deliberations. In each case, statistical sampling will permit defensible results within preselected limits of precision and confidence.

The discussion that follows is not intended as a criticism of the audit which was performed or the validity of its stated objectives. Nor is it implied that alternate plans presented have universal application in every audit situation.

#### ALTERNATE PLAN A

Audit Objective: To find the "error" rate for each location and for the 12 locations combined within preselected limits of precision and confidence.

Considerations: The number of items to be audited at each location (sample size) will be determined by the worse expected error rate, the size of the universe, 95 percent confidence, and precision consistent with the location error rate and audit requirements.

Method:

A preliminary random sample at each location will be used to estimate the error rate. The error rate found is to be used with the desired precision and confidence to determine the final sample size.

The actual audit plan provided uniform criteria for use at all locations for determining sample sizes. Application of the uniform criteria resulted in an overall sample size of 4,056. Alternate Plan A, however, differs from the actual audit plan in that sample sizes are to be determined for each location based on the results of a preliminary sample.

As may be noted from Table A, the actual findings at most of the locations were considerably different than the uniform criteria used to determine sample sizes. The actual findings ranged from a low of 1.4 percent to a high of 48.6 percent and only those findings at locations 1, 8, and 9 were close to the  $10\% \pm 3\%$  uniform criteria used for determining sample sizes. In this situation, the use of uniform criteria unnecessarily increased the amount of the overall time needed to perform the audit.

For instance, if the auditor determines that a finding of  $10\% \pm 3\%$  at an individual location is adequate for the audit purpose, he may also believe that a finding of  $40\% \pm 12\%$  is equally adequate. For location 7 (see Table A), the application of the  $10\% \pm 3\%$  criteria resulted in a sample size of 372 items. However, a preliminary random sample would have probably revealed an error rate of approximately 40 percent (actual error rate was 36.6 percent). If the auditor considered a finding of  $40\% \pm 12\%$  adequate, the sample size at location 7 could have been reduced from 372 to 64.

Alternate Plan A is based on the above except that the desired sampling precision was not increased in proportion to the expected error rate i.e.;  $10\% \pm 3\%$ ;  $20\% \pm 6\%$ ;  $30\% \pm 9\%$ ;  $40\% \pm 12\%$ . By referring to Table B it can be



seen that the expected error rate and sampling precision for location 8 is 10%  $\pm$  3% while the expected error rate and sampling precision for location 7 is 40%  $\pm$  7%. The sampling precision (tolerable error) is an audit judgment and will depend upon the degree of precision which the auditor believes will be needed to convince agency officials and other readers of our reports of the need to take necessary action to correct the situation being reported on. Certainly, there should be very few instances where we find it necessary to obtain sample sizes large enough to provide sampling precision of plus or minus 3 percent when the error rate is 40 percent.

Implementation of Alternate Plan A requires that the following (or similar) instructions be included in the audit program:

1. At the assigned location, select at random a preliminary sample of 150 items from the universe subject to audit.
2. The items in the preliminary sample are to be audited in accordance with the instructions set forth in the audit program.
3. When the preliminary sample audit is completed, calculate the error rate (number of errors found/150).
4. Round the resulting error rate to the nearest higher 5 percent interval. (e.g., if resulting error rate is 11.4%, round to 15%; if 18.2%, round to 20%, if 23.8%, round to 25%; etc.)
5. Select the final sample size from the appropriate sample size table or by use of a formula if appropriate sample size tables are not available. (See Audit Sampling Memo 7-19 for an example of a calculation of a sample size by using the sample size formula.)

The audit program should make it clear that the preliminary sample selected and audited is part of the final sample. When the final sample size has been determined (e.g., 260), only the number of additional items (110) needed to bring the preliminary sample size (150) up to the final sample size (260) are to be selected and audited. Results from the audit of the additional items are then combined with results previously obtained from the preliminary sample items to arrive at the final sample results for the location universe.

Under Alternate Plan A, preliminary error rate findings, based on samples of 150, would not vary appreciably from the audit results obtained for each location during the actual audit.

The most likely sampling plan, resulting from Alternate Plan A, is summarized below in Table B.

T A B L E B

<u>Location</u>	<u>Number in Universe</u>	<u>Sample Size Criteria<sup>a</sup></u>		<u>Sample Size Required</u>	<u>Sample Size Actually Used</u>
		<u>Worse Rate of Occurrence (preliminary)</u>	<u>Tolerable Error With 95% Confidence</u>		
1	4,183	15%	±4%	290	370
2	4,498	20	5	230	356
3	2,350	5	2.5	260	318
4	19,721	35	6	240	377
5	14,145	35	6	240	375
6	3,525	50	7	190	354
7	14,705	40	7	190	372
8	5,148	10	3	360	328
9	4,366	15	4	290	308
10	2,810	5	2.5	270	355
11	913	25	5	220	352
12	190	-	-	190	190
				<u>2,970</u>	<u>4,055</u>

<sup>a</sup>Preselected based on acceptable tolerable errors for the varying preliminary error rate findings.

Alternate Plan A reduces the overall number of sample items needed from 4,056 to 2,970. With approximately 1,100 fewer items to audit, more than 25 percent of the work in this area is saved.

Assuming that error rate findings remain the same, Table C below compares the sampling precision provided by Alternate Plan A with the precision actually obtained in the audit.

T A B L E C

<u>Location</u>	<u>Error Rate</u>	<u>Tolerable Error with 95 Percent Confidence</u>	
		<u>Actual</u>	<u>Plan A</u>
1	11.9%	±3.2%	±3.6%
2	15.7	3.6	4.6
3	4.7	2.2	2.4
4	33.7	4.7	5.9
5	32.8	4.7	5.9
6	48.6	4.9	6.9
7	36.6	4.8	6.8
8	8.2	2.9	2.7
9	10.1	3.2	3.4
10	1.4	1.1	1.3
11	20.1	3.3	4.6
12	<u>17.4</u>	<u>-</u>	<u>-</u>
	27.2%	±1.8%	±2.3%

The actual audit initially committed manpower to select and audit 4,056 sample items. Alternate Plan A, by specifying preliminary samples of 150 at each location (1,800 overall), initially commits less than half of the budgeted time for selection and audit. Preliminary sample results then provide the necessary information to select sample sizes at each location consistent with the "worse expected error rate" and acceptable levels of sampling precision.

ALTERNATE PLAN B

Audit Objective: To find the overall "error" rate, within prescribed limits of precision and confidence, for the universe of 76,554 items at 12 different agency locations.

Considerations: For audit purposes, the universes at the individual locations lose their individual identities. Under Alternate Plan B, each location universe is important only to the extent that it is a part of the overall universe of interest.

Method: The overall final sample size will be selected to provide acceptable limits of precision and confidence. The number of sample items for each location will be selected on a proportionate allocation basis.

Alternate Plan B differs from the actual audit and Alternate Plan A in that defensible sampling results are specified only at the overall level.

The overall final sample size must be sufficiently large to provide the predetermined precision and confidence acceptable to the responsible audit group.

The physical location of the 12 agency sites makes it desirable to avoid the use of a preliminary sample approach for arriving at a final sample size. With unequal parts of the overall universe at 12 different locations, preliminary sampling would result in considerable delays while individual location audit teams completed assigned preliminary sample quotas and made results available for final sample size determination.

Fortunately, statistical sampling provides a "back door" approach for arriving at overall final sample size in keeping with acceptable levels of desired sampling accuracy.

Tables can be prepared for guidance in selecting the final sample size which will provide acceptable levels of precision and confidence regardless of the eventual error rate disclosed by the audit.

Table D below, for example, shows the sampling precision, with 95% confidence, for error rate intervals of 5%, based on sample sizes of 300, 400, and 500 from a universe of 76,554.

T A B L E D

<u>If Overall Error Rate Finding is:</u>	Sampling Variability with 95% Confidence is:		
	<u>For Sample of 300</u>	<u>For Sample of 400</u>	<u>For Sample of 500</u>
5%	± 2.5%	± 2.1%	± 1.9%
10	3.4	2.9	2.6
15	4.0	3.5	3.1
20	4.5	3.9	3.5
25	4.9	4.2	3.8
30	5.2	4.5	4.0
35	5.4	4.7	4.2
40	5.5	4.8	4.3
45	5.6	4.9	4.4
50	5.7	4.9	4.4

Based on the actual audit finding of 27.2% overall, the precision provided by the sample sizes shown is as follows:

27.2%	± 5.0%	± 4.4%	± 3.9%
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The above table, which can easily be expanded to include any sample size, allows the audit group to exercise the necessary judgment for selecting a sample size consistent with the audit objective as to the desired precision of sampling results.

The audit group need only select the sample size that provides the acceptable precision regardless of the error rate finally disclosed by the audit.

The Plan B audit objective and sampling plan preclude statistically reliable findings at the individual location level. Assuming an overall sample size of 500, sampling quotas for individual locations would be too small for valid findings.

However, Plan B reduces the audit sample size needed from 4,056 (actual audit) or 2,970 (Plan A audit) to 300, 400, or 500 depending on the desired precision.

The savings in audit time, therefore, are very significant especially if audit time per sample item is considerable.

In any case, the audit group should weigh the need and contribution of statistically reliable results at the location level against the increased costs of obtaining them.

Actually, the final report, under Plan B, can include individual location findings provided that proper language is used and no statistical significance is attached to the results at the individual locations.

#### ALTERNATE PLAN C

Audit Objective: To find the overall Agency error rate in a universe of 1,280,000 items. The agency comprises 200 widely dispersed locations each of which contains an unequal part of the total item universe.

Considerations: A simple unrestricted sampling plan, requiring a random selection of items from the entire universe, could be used.

If all universe items were physically located in the same place, this would probably be the easiest and best approach.

In this situation, however, unrestricted random selection would undoubtedly result in sample items being chosen from all or most of the 200 agency locations.

Travel costs incurred, due to the need to conduct very small audits at all or most of the agency locations, could make this approach prohibitively expensive.

Method: Two-stage random sampling for attributes.

The plan is "two-stage" because sampling is required at two separate and distinct levels.

A sample of primary units (individual agency locations) is first selected at random, and from the selected sample of primary units, a random sample of secondary units (items of audit interest) is taken.

For the purposes of this audit, the first stage universe is the 200 individual agency locations, and the second stage universe is the 1,280,000 items of audit interest.

The actual audit and Alternate Plans A and B limited the universe of interest to the 76,544 items at the 12 selected agency locations. Alternate Plan C differs from the actual audit and other plans presented in that the error rate for the entire agency (200 locations) is what is of interest rather than the error rate of just 12 of the 200 locations.

Our professional staff will probably find that assistance will be needed in devising a two-stage sampling plan. Also, assistance will probably be needed in interpreting the results of such a sampling plan. This assistance can be obtained from the statistician in the Office of Policy and Special Studies.

#### Alternate Plan C - Sampling Procedure and Audit Results

Sampling sizes used for implementation of Alternate Plan C are as follows:

First Stage Sample =	20 (agency locations)
Second Stage Sample =	75 (items from first stage universes)

Table E below summarizes the sampling plan and audit results for the 20 randomly selected agency locations in the first stage sample.

Results for locations 1 through 12 are essentially the same as those found in the actual audit. (See Table A.) Universes have been rounded and error rates reflect the slight changes necessitated by sample sizes of 75. Locations 13-20 are hypothetical but consistent with error rate findings and universe sizes of the 12 locations actually audited.



T A B L E E

Sample Location	Number in Universe	Number in Sample	Sample Items With Error		Estimated <sup>a</sup> Universe Items With Error
			Number	Percent	
1	4,000	75	9	12.0%	480
2	4,500	75	12	16.0	720
3	2,500	75	4	5.3	133
4	20,000	75	26	34.7	6,940
5	14,000	75	25	33.3	4,662
6	3,500	75	36	48.0	1,680
7	15,000	75	28	37.3	5,595
8	5,000	75	6	8.0	400
9	4,500	75	8	10.7	482
10	3,000	75	1	1.3	39
11	1,000	75	15	20.0	200
12	500	75	13	17.3	86
13	13,000	75	17	22.7	2,951
14	2,000	75	21	28.0	560
15	5,000	75	26	34.7	1,735
16	6,000	75	20	26.7	1,602
17	10,000	75	31	41.3	4,130
18	7,000	75	2	2.7	189
19	1,000	75	15	20.0	200
20	<u>5,000</u>	<u>75</u>	<u>24</u>	<u>32.0</u>	<u>1,600</u>
	<u>126,500</u>	<u>1,500</u>	<u>339</u>	<u>27.2<sup>b</sup></u>	<u>34,384</u>

<sup>a</sup> Column figures arrived at by projecting sample error rate findings to total items in location universes. (e.g., for location 1; 12% of 4,000 = .12(4,000) = 480)

<sup>b</sup> 27.2% is the weighted overall error rate for the 20 locations in the first stage sample.

Note: The overall error rate of 27.2% is calculated as follows:

$$\frac{\text{total est. universe items with error}}{\text{total of 20 location universes}} = \frac{34,384}{126,500} = .272 \text{ or } 27.2\%$$

In this sampling situation, the overall error rate is not the arithmetic average of the individual location error rates nor can it be calculated by dividing the total errors found in the sample (339) by total items in the second stage sample (1,500).

Both these methods result in an overall error rate of 22.6% which is incorrect because it is unweighted.

If proportionate allocation had been used in assigning second stage sample sizes, either of the two methods above would give the correct overall error rate because the individual location results would be self-weighting.

## Sampling Variability

The formula for calculating the sampling variability for two-stage sampling for attributes is rather complex. It takes into account and combines the contribution to sampling variability from each stage of sampling.

For the 27.2% error rate finding under Plan C, the sampling variability, with 95% confidence, works out to be  $\pm 6.0\%$ .

This variability, for the Plan C example, is abnormally high due to the wide variation among individual location error rates. In the actual audit, individual location error rate findings ranged from 1.4% to 48.6% which is about maximum variation in sampling for attributes.

(In sampling for attributes, a 50% "error" rate is maximum in that it results in the largest sampling variability. This is so because error rate findings of 60% have the same variability as 40%, 70% as 30%.....99% as 1%.)

If the Plan C sampling procedure was unchanged and individual location error rates were between 5% and 20%, the sampling variability would be reduced to  $\pm 1.8\%$  with 95% confidence.

In two-stage sampling for attributes, the number of items in the second stage sample, from each location, have very little effect on overall sampling variability.

Thus, if second stage sample sizes used in Plan C were increased from 75 to 500 or even 1,000, the overall sampling variability of the 27.2% finding would only be reduced approximately two-tenths of one percent. However, with second stage sample sizes at 75, an increase of first stage locations from 20 to 40 would reduce the sampling variability to  $\pm 3.6\%$ .

In two-stage sampling for attributes, therefore, the number of locations in the first stage sample has a much greater effect on sampling variability than the number of

items in the second stage samples providing they are sufficiently large.

### Estimating the Total Universe Items with "Error"

The 27.2% error rate finding in the 126,500 item universe of the 20 sampled locations is assumed to hold true for the entire 200 agency location universe of 1,280,000 within calculated limits of precision and confidence.

Therefore:

Total Estimated items with		
"Error"	= 1,280,000(.272) =	348,160
Sampling Variability	= 1,280,000(.060) = $\pm$	76,800

### Summary - Plan C

Plan C, with overall sample size of 1,500, representing 75 sample items at each of 20 agency locations, permits the audit group to project findings agency-wide.

Plan C sampling procedure is equally valid for finding the error rate and sampling variability for one line item of data or many line items of data on the same sample document.

The sampling variability found in the example is unusually high due to the extreme variation in individual location error rates. It is more "normal" for a two-stage sampling plan, as used in this example, to produce a sampling variability of approximately  $\pm 2\%$ .

Two-stage sampling requires that first stage locations be selected at random. This requirement may be inconsistent with available regional office manpower.

OVERALL SUMMARY

Actual Audit and Alternate Plans, A, B, C

	Audit Objective	Audit Universe	Procedure Used	Overall Sample Size	Time Saved <sup>a</sup>
Actual Audit	To find "error" rate within acceptable limits of precision and confidence at each of 12 locations and overall.	12 locations of varying size universes containing 76,544 items of audit interest.	Individual location sample sizes determined from "tables". Sampling criteria (10% ± 3.0% - 95% confidence) uniformly applied at 11 of 12 locations audited.	4,056	-
Plan A	(same as above)	(same as above)	Individual location sample sizes based on results of preliminary samples of 150. Final sample sizes based on preliminary error rate findings and acceptable precision and confidence for each location.	2,970	1,086 units
Plan B	To find "error" rate within acceptable limits of precision and confidence at the overall (12 locations combined) level.	The 76,554 items of audit interest at the 12 agency locations.	Overall sample size determined from precisions calculated for various sample sizes and for each 5% interval of possible worse error rate.	300, 400, or 500	3,556 units <sup>b</sup>
Plan C	To find overall agency "error" rate. (200 locations)	The 1,280,000 items of audit interest at 200 agency locations.	Two-stage sampling: 20 first stage locations 75 second stage sample items	1,500 and 20 locations	2,556 units <sup>c</sup>

<sup>a</sup>Based on average time units, i.e., the average amount of time required to audit one sample item. Experience has shown that audit time per item can vary from several minutes to a full day or more depending on the nature of the audit, back-up documents required, number of line items examined, etc. Does not include time spent selecting and locating sample items.

<sup>b</sup>Based on overall sample size of 500.

<sup>c</sup>Time saved is partially offset by the requirement to audit 8 additional locations.