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Spatial Modeling of Production Capacities

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Introduction

Stated generally, industrial location theory says that industry output by location is a function of profits. Most empirical models that try to explain industry location use either output or changes in output as the dependent variable, and profit proxy variables as independent variables. In this paper we argue that new capacity output--output associated with fully utilized gross investment-in-place--is a more appropriate dependent variable. Capacity output would be the desired level of output, given the capital stock, but the actual output in any one period may not be at the desired level because of unanticipated economic conditions. Also we argue that the profits associated with locating a marginal unit of output is the appropriate explanatory variable. After presenting the arguments to support our positions we show the results of applying such a model to 59 manufacturing industries.

The theoretical background for the model that follows is the standard theory of the firm. Firms choose production plans that maximize the estimated present value of profits, given current prices and expectations about future prices. The plans include the amounts of outputs, intermediate inputs, capital stock and the locations of the facilities.

Firms place capital stock in locations that maximize their profits. A new firm is concerned with locating a new plant; but most production decisions are made by existing firms that have existing capital stock sunk in particular locatins. Their concern is with locating increments to capital stock, which may involve the expansion of an old plant rather than the construction of a new plant. We assume that business decisions in any one period are short-run decisions which modify existing production plans. If a firm is thinking about relocation, the profits at any new location must be higher than the sum of profits at the old location and the fixed costs at the old location. Thus the existing capital stock has to be taken into account.

Deriving the New Capacity Output

Although some of our discussion will still be about firms all of the variables used in the model are for industries and regions. That is, we will talk about the location decisions of the firm, but our model explains the location of industries within regions, independent of the number of firms that may be in any one industry and region.

Each period some capital stock is lost because of depreciation (including obsolesence) and gross investment may be added to both replace the depreciated capital and add to the capital stock. Thus:

$$K_g^t = K_g^{t-1} - D_g^{t-1} + I_g^t \tag{1}$$

where: K is capital available for production for one industry.

D is depreciation.

I is gross investment that is in place and available for use.

t denotes time period, and

g denotes region.

Since the construction/installation time may be more than one production period a distinction is being made between capital goods purchases and capital-in-place. The variable I_g^t refers to new capital-in-place in time t, region g and <u>not</u> capital purchases in t, region g.

Assuming that all capital is fully utilized, we can rewrite (1) in terms of capacity output by applying the appropriate output-capital ratios to the capital stock. The new equation is:

$$KQ_g^t = KQ_g^{t-1} - DQ_g^{t-1} + IQ_g^t \qquad (2)$$

where: KQ is the output associated with fully utilizing capital stock (capacity output).

DQ is the output associated with the depreciated capital stock.

IQ is the output associated with the gross investment put in place during the period (new capacity output).

We let the depreciated output be equal to the depreciation rate times the actual output since economic depreciation is a function of the use of capital stock. Thus:

$$DQ_g^t = d_g^t Q_g^t \qquad (3)$$

where: d is the depreciation rate, and

Q is the level of actual output.

Also, we define the capacity output as being equal to the actual output devided by the capital utilization rate:

$$KQ_g^t = Q_g^t/u_g^t \tag{4}$$

where: u is the capital utilization rate.

By substituting equations (3) and (4) into (2) and solving for IQ we obtain the definition:

$$IQ_g^t = Q_g^t/u_g^t - Q_g^{t-1}/u_g^{t-1} + d_g^{t-1}Q_g^{t-1} \tag{5}$$

The location decisions of the firms within an industry involve selecting IQ_g^t for each period and region. We are saying that the ability to produce in any region where there is production will decrease because of depreciation of capital stock, and that the decision each year is where to locate the output associated with gross investment. The IQ can be located either in the same regions or in different regions. If it is in the same regions it is being used first to replace the loss of output due to the depreciated capital stock. If the existing plants are in low profit regions, the IQ will be located in other higher profit regions, but production may still take place in the older regions since they may still have usable capital stock on which rental payments have to be made.

Deriving the Profit Variable

The general theory is that output is located in response to profits and we have argued above that the determination of capacity output associated with the gross investment is the appropriate output variable in location decisions. In this section we are concerned with what is the appropriate measure of profits. What we should have is a marginal concept of locational profits; that is, the profits that would be associated with an additional unit of capacity output. This concept of marginal profits can be illustrated with a one market, Von Thunen example.

Assume that there is one market region h supplied by a number of producing regions g (g = 1, ..., n). After the market has cleared, there is information on how much was supplied by each producing region, and the per unit cost of producing and transporting from g to h. The producing region with the highest cost is the one that establishes the market price. Thus, the profits for producing regions are:

$$\Pi_g = P_h - C_{gh} \qquad g = 1, \ldots, n \tag{6}$$

where: P_h is the price in market region h,

C_{gh} is the cost of producing goods at g and transporting them to h, and

Π_g is the marginal location profit in region g.[1]

Since the marginal supplier establishes the market price there are no profits for the marginal supplier, which we denote as m. Therefore:

$$P_h = C_{mh} \tag{7}$$

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[1]There is an implicit time superscript t in (6) and most of the remaining equations which is being left out for convenience.

and by substituting (7) into (6) we derive:

$$\Pi_g = C_{mh} - C_{gh} \qquad g = 1, \ldots, n \qquad (8)$$

When there are many markets this same concept of locational profits can be derived using linear programming. At equilibrium, each selling location has maximum profits and each buying location purchases good at the lowest possible price. The objective in the linear program algorithm is to minimize total cost of production and transportation; that is:

$$\Phi = \sum_g \sum_h C_{gh} X_{gh} \qquad g,h = 1, \ldots, n \qquad (9)$$

subject to:

$$\sum_h X_{gh} = S_g \qquad g = 1, \ldots, n \qquad (10)$$

$$\sum_g X_{gh} = D_h \qquad h = 1, \ldots, n \qquad (11)$$

$$X_{gh} \geqq 0 \qquad g,h = 1, \ldots, n \qquad (12)$$

where: X_{gh} is the amount of goods shipped from region g to region h.

S_g is the amount of supply of goods in region g, and

D_h is the demand for goods in region h after the market has cleared.

We are using S for supply here instead of Q for output since the supply of goods entering the market from region g may be different than the output in region g because of foreign imports or changes in inventories. When (9) is minimized, the following condition holds:

$$P_h - \Pi_g < C_{gh} \qquad g,h = 1,\ldots,n \qquad (13)$$

and when shipments between g and h are positive:

$$P_h - \Pi_g = C_{gh} \qquad \text{if } X_{gh} > 0 \qquad g,h = 1, \ldots, n \qquad (14)$$

The Π_g and the P_h in (13) and (14) are shadow prices on constraints (10) and (11) respectively. If an additional unit of output were to be produced at region g, the profits for that additional unit would be Π_g; thus we refer to Π_g as marginal location profits.[2]

A Partial Measure of Location Profits

Research is underway to have a complete measure of location profits as described above, but in this paper we can only report on results that use a partial measure of location profits. We will show what is contained in this partial measure.

First we look at the components of C_{gh}. They are:

$$C_{gh} = L_g + \sum_i a_g^i P_g^i + E_g + T_{gh} \qquad g,h = 1, \ldots, n \qquad (15)$$

where: L is the labor cost per unit of output.

a^i is the input-output coefficient -- the amount of input i per unit of output used by the producing industry.

E is the other expenses (including land) per unit of output, and

T_{gh} is the transportation cost of shipping a unit from region g to region h.

[2]See Stevens [1961] for interpretation of the shadow prices.

Substituting (15) into (8) we obtain for the one market problem a complete measure of location profits with all of its components:

$$\Pi_g = (L_m - L_g) + \sum_i (a_m^i P_m^i - a_g^i P_g^i) + (E_m - E_g) + (T_{mh} - T_{gh}) \quad g=1,\ldots,n \qquad (16)$$

Each term in parenthesis on the right-hand side of (16) is a component of location profits; that due to labor costs, material costs, other expenses, and transport cost, respectively.

Next, we note that when the objective function (9) is minimized, the only variation in the market prices is due to transportation costs; since if there were no transportation costs, the prices would be the same in all markets. Therefore, any price i could be expressed in two components such as :

$$P_g^i = V_g^i + K^i \qquad g = 1, \ldots, n \qquad (17)$$

where: V is a variable that has a positive value because the variation in transportation costs, and

K is a constant.

We now modify (16). Firstly, we assume that the input-output coefficients are the same in all regions; therefore the material cost component of the location rent would have variation only because of the variable V. Secondly, we drop the component for other expenses since we don't have the data. The partial location profits variable is defined as follows:

$$LR_g = (L_m - L_g) + \sum_i a^i (V_m^i - V_g^i) + W_g \qquad g = 1, \ldots, n \qquad (18)$$

where: W represents the location profits associated with transportation costs when there are many markets,[3] and

LR is the partial measure of location profits.

It has been shown by Nadji and Harris [1983] that the V's and the W's are shadow prices computed when the transportation cost T_{gh} replaces C_{gh} in the objective function (9). Thus, the location profits LR can be estimated for every commodity by deriving the V's and W's from the linear programming algorithms that minimize transportation costs, with data on payrolls per unit of output, and with national input-output coefficients. The LR variable accounts for regional variation in labor and transport costs.

The Relationship between New Capacity Output and Profits

So far we have argued that the appropriate function for short-run location decisions is that new capacity output is a function of marginal locational profits. Thus:

$$IQ_g = f(\Pi_g) \tag{19}$$

The questions now are: what does this function look like and how should the parameters in the equation be estimated?

One procedure would be to allocate all the new capacity output to the region which has the highest marginal location profits. However, this would not be realistic since with casual observation we know that gross investment is not located all in one region in any one year. The gross investment comes in

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[3] We can't use $T_{mh} - T_{gh}$ with many markets since the marginal region m may not ship to region h.

discrete units and if the first unit were to be allocated to the highest profit region, all the prices in the system would change, and the new location profits may indicate that the region with the highest profits is different. If we were to allocate the first unit to the highest region and recompute profits, allocate the second unit to the new highest region and so forth; then over the period of a year, because of the changing prices, many regions would receive some of the new capacity otuput.

Also gross investment is spread out over many regions because different types of investment have different useful lives. For example, the life of a plant is much longer than the life of a machine. Firms may install new equipment in existing plants even when the profits are low at those locations because there is no existing plants in the higher profit locations and because fixed rental payments are due still on the old plants. A similar issue is that there is a threshhold problem due to the discretness of new plants. Additional capacity may be added to an existing plant in an old location rather than build a new plant in a higher profit location since the minimum-sized new plant is too large.

In (19) we related new capacity outputs and profits in the same time period, but this may not be appropriate if the construction/installation period is larger than the production period. The decision to purchase the capital necessary to have investment-in-place this period must have been made before the current period. Thus, the output associated with gross investment-in-place may be a function of profits of a previous period or some weighted average of previous periods.

We chose to fit function (19) modified because we only have a partial measure of profit, using pooled cross-sectional and time series data. Essentially, the parameters in our estimation would be cross-sectional parameters since we have 585 regions and only five years. With cross-sectional data, a regression equation would assign the highest amounts of new capacity output to the region with the highest marginal location profits; but other regions with high profits would also receive some of the new capacity output.

The Results

Using data for 585 regions[4] and five years (1970-1974) the following equation was estimated for 59 manufacturing industries:

$$IQ_g^t = F(\Delta S_g^{t-1}, LR_g^{t-1}, VL_g^{t-1}, IS_g^{t-1}, \Delta D_g^{t-1}, D_g^{t-1}) \tag{20}$$

where: Δ denotes a change from year t-2 to year t-1,

VL is the value of agriculture land per acre, and

IS is the amount of inputs imported into a region per unit of output, which we call "input scarcity."

Since there is only a partial measure of location profits the variables in (20) other than LR, serve as proxies for determinants of location profits not accounted for in LR. The variables ΔD, D, and ΔS pick up the agglomeration effects that competitors and buyers have on revenues and production costs. The input scarcity variable acts like a weighted average of input prices since the greater the amounts of inputs imported, the higher the cost of acquiring the inputs. The regional variation in input prices would be accounted for completely with the shadow price V of (18) if in fact the objective function

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[4]SMSA's and rest-of-BEA economic areas, both subdivided by state when necessary.

(9) were minimized. However, in the real world the presence of transport cross-hauls indicates that (9) is not minimized; therefore, there would be some regional variation in K of (17). The variation of the input scarcity variable serves as a proxy for the variation on input prices not accounted for with the V's.

All of the data used to estimate (20) came from the Regional Forecasting Project at the University of Maryland. The original data sources and data estimating procedures are given in Harris and Nadji [1983]. A few words need to be said, however, about the estimation of the IQ's, d's and u's for regions, since as many readers are aware, there are no published sources for this information.

First, gross investment-in-place (I) was estimated for the nation and regions by assuming that the capital good purchases in any year t were put into place and available for use in production during the year t, t+1, and t+2. In other words, $I^t = \alpha^t \, IP^t + \alpha^{t-1} \, IP^{t-1} + \alpha^{t-2} \, IP^{t-2}$, where IP is the investment purchases and the sum of the α weights is equal to one.

IQ was estimated for each region and industry by assuming that the relationship between IQ and I was the same for all regions; that is, it was assumed that the productivity of gross investment is the same everywhere since all investors have access to the latest technology. IQ was first computed with (5) at the national level and the national IQ/I ratio applied to the regional I's.

Data on the capacity utilization rates (u) and the depreciation rates (d) are available at the national level, and we estimated the u's and d's for states using (5). The first step was to normalize the national u's for each industry by dividing each year's value by the peak year value during the historic period 1970-1975. We assume that a state's u cannot exceed one, therefore, the u's in all states would be equal to one in the peak year. This assumption reduced the number of unknowns in (5) to two, namely: d_g^{t-1} and u_g^{t-1} when t is the peak or pre-peak year, and d_g^{t-1} and u_g^t when t is a post-peak year. Working separately for each industry in both directions from the peak year, the unknown d's and u's were estimated with a matrix balancing procedure that made use of the fact that the state values of each component of (5) must add to its national value.

The regression results for the 59 manufacturing industries are given in Table 1. After a correction for heteroscedesticity the variables were entered into an ordinary least-squares program in the order given in Table 1 with a restriction that if the coefficient of an entering variable had the wrong sign or made the coefficient a previously entered variable having the wrong sign, the entering variable was rejected. The steps were recycled so that if a variable were rejected and a following variable were accepted, the procedure would try to re-enter the rejected variable before proceeding to the next variable on the list.

The coefficients for variables VL and IS were restricted to be negative and LR, ΔD and D were restricted to be positive. The variable ΔS was allowed to take either sign. All regional variables except VL and IS were entered in the equation as shares of their national totals. In addition to the usual coefficient of determination, which is labeled $\bar{R}^2_{IQ}$ in the table, we computed $\bar{R}^2_{KQ}$ which is a measure of how well (20) was able to predict KQ_g^t, which is computed using (2) with the predicted IQ_g^t on the right-hand side. $\bar{R}^2_{KQ}$ would be the appropriate measure of overall fit in most applications, such as in models to forecast levels of production.

The results given in Table 1 show that $\bar{R}^2_{KQ}$ is high for all industries and that most of the coefficients are highly significant. The frequency and significance of the location profit variable, LR, strongly supports our theoretical model.

References

Harris, Curtis C. and Mehrzad Nadji (1983), Regional Economic Forecasting: Theory and Practice, (forthcoming).

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Stevens, Benjamin H. (1961), "Linear Programming and Location Rent," Journal of Regional Science, Vol. 3, No. 2.

Table 1
Equations Explaing Capacity Output Associated with
Gross Investment In-Place (IQ) by Industry

Industry	SIC(1967)	VARIABLES ΔS	LR 10^{-1}	VL 10^{-2}	IS 10^{-4}	ΔD	D 10^{-1}	$\bar{R}^2_{IQ}$	$\bar{R}^2$
Ordnance	19	-.54*	16.87#			.54+		.26	.8
Meat Packing	201	-.21#	.33#	-.23*	-.19+			.11	.9
Dairy Products	202	-.29#	.21#		-.23#		.46#	.29	.9
Canned & Frozen Food	203	-.19#	.03			.08*	.05	.07	.9
Grain Mill Products	204	-.23#	.25#	-.35*				.37	.9
Beverages	208	-.11#	.27#	-.33			.31#	.23	.9
Misc. Food Products	205,206,207,209	-.29#	.04#					.07	.9
Tobacco Products	21	-.02#	.30#				.06	.22	.9
Fabrics & Yarn	221,222,223,224,226,228	-1.22#	.05	-.69*				.86	.9
Misc Textiles	227,229	-.21#	.55#	-.33		.04		.05	.9
Apparel & Knitting	225,23,239	-.26#	.08				.00	.12	.9
Misc. Fabricated Textiles	239	-1.72#	2.73#	-11.49#	-5.97#		1.48#	.98	.9
Lumber & Wood Products	24	-.28#	.09#	-.50#				.54	.9
Furniture & Fixtures	25	-.15#	.07#			.00		.09	.9
Pulp & Paper Mills	261,262,263	-.06#	.26#	-.30#	-.42#			.29	.9
Paper Products	264,265,266	-.03	.45#	-.52+		.62#	.02	.31	.9
Printing & Publishing	27	.13#	.23#	-1.94#	-.47#	.48#	1.35#	.86	.9
Industrial Chemicals	281	.22#	.74#	-1.94#				.86	.9
Plastics & Synthetics	282	-.16#	.45#					.81	.9
Drugs	283	-.45#	.37+	-1.59*	-.82	.03		.20	.9
Cleaning & Toilet Prep.	284	.04	.43#			.30#	.16#	.41	.9
Paints & Allied Prod.	285	-.07#	.22#			.02		.29	.9
Agricultural Chemic.	287	-.20#	.71#				.24#	.93	.9
Misc. Chemicals	286,289	-.16+	.08			.77#	.61#	.31	.9
Petroleum Refining	29	-.47#	.16#	-3.32+			.25#	1.00	.9
Tires & Tubes	301	-.03	.29#	-.02*		.09#	.05+	.16	.9
Misc. Rubber Products	302,303,306	.25#	1.03#	-.14#	-4.83#		1.00#	.80	.9
Plastic Products	307	.68#	1.99#					.97	.9
Leather & Leather Prod.	31	-1.12#	.68#				.10#	.77	.9
Stone,Clay & Glass Prod.	32	-1.13#	.12#			.17#		.71	.9
Iron & Steel	331,332,3391,3399	-.04	.46#	-2.22#		.01		.70	.99
Copper	3331,334,3351,3362	.49#	.23#				.79#	.29	.9

Table 1 (Continued)
Equations Explaining Capacity Output Associated with Gross Investment In-Place (IQ) by Industry

Industry	SIC(1967)	VARIABLES ΔS	LR 10^{-1}	VL 10^{-2}	IS 10^{-4}	ΔD	D 10^{-1}	$\bar{R}^2_{IQ}$	$\bar{R}^2_{KQ}$
Aluminum	3334,3352,3361	-.19#	1.11#	-.02#				.21	.94
Misc.Non-Ferrous Met.	3332,3333,3339 3356,3357,3369,3392	-.29#	.36#				.12+	.11	.79
Metal Containers	341,3491	.14*	.52*			.55#		.07	.95
Heating,Plumbing,Stamping & Screen Products	343,345,346	-.29#	.88#		-1.18#	.66#		.57	.99
Structural Metal Prod.	344	.21#	.54#		-1.27#	.24#	.56#	.77	.94
Misc.Fab.Metal Prod.	342,347,348,349,-3491	.08+	.23#	-.34#		.08#		.50	.98
Engines & Turbines	351	-.57#	1.06#			.04		.92	.95
Farm Equipment	352	.20#	.18#	-.07			.14#	.30	.97
Construction Mining Equip.	353	-.60#	.09#				.23#	.43	.96
Metal Working Machinery	354	.14#	.06#					.08	.98
Industrial Machinery	355,356	-.31#	.57#	-1.59+		.28#	.71#	.08	.95
Office & Computer Mach.	357	-.62#	1.17#		-4.63#			.96	.96
Service Industry Mach.	358	-.13#	.35#	-.39*		.17#	.26#	.59	.94
Misc. Machinery	359	-.43#	.11#					.27	.96
Elect. Apparatus&Transmis.Equip.	361,362	-.26#	.76#	-1.63#	-.60#			.93	.94
Household Appliances	363	.03	.10*		-.36+	.00	.13#	.14	.95
Elect. Lighting&Wiring Equip.	364	1.77#	.65#			.00		.89	.88
Radio,TV&Communication Equip.	365,366	-.43#	.46#	-.80#		.37#		.53	.93
Electronic Components	367	-.93#	1.84#		-4.52#	.05	.44+	.73	.92
Misc. Electrical Items	369	-1.30#	.92#			.06		.82	.96
Motor Vehicles	371	-.06	.12#	-.04		.03		.06	.98
Aircraft & Parts	372	-.88#	.17+			.40#	1.26#	.48	.94
Railroad Equipment	374	-.44#	.63#	-.43		.15	.44#	.63	.97
Misc. Transportation Equip.	373,375,379	-.89#	.43#	-7.77#		.68#	.41#	.52	.85
Scientific&Medical Instrum.	381,382,384	-1.73#	3.61#	-14.15#		.85#		.92	.90
Optical,Photo Eqp.&Clocks	383,385,386,387	-1.37#	.78#	-7.22#	-1.34+	.87#	1.30#	.99	.97
Manufacturing	39	-.10+	.15#			.54#	.05	.42	.95

* denotes significance at .10 level
\+ denotes significance at .05 level
\# denotes significance at .01 level

ΔS denotes one year change in supply (000,1976$)
LR denotes location profits (partial measure)
VL denotes value of agricultural land per acre

IS denotes input scarcity
ΔD denotes one year change in demand (000,1976$)
D denotes demand (000,1976$)