DIVISION OF FINANCIAL AND
GENERAL MANAGEMENT STUDIES

This case study describes a GAO audit staff's use of the chi-square test for independence. The test was part of the analysis to evaluate the Law Enforcement Education Program. Our intent in issuing this booklet is to illustrate how the chi-square test can contribute to GAO's work and to help auditors recognize when it can be used.

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AN AUDIT USE OF THE CHI—SQUARE TEST FOR INDEPENDENCE:

A CASE STUDY

Financial and General Management
Studies Division

United States General Accounting Office

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CHAPTER I

INTRODUCTION

This case study describes a GAO audit staff's use of the chi-square test for independence. The test was part of the analysis to evaluate the Law Enforcement Education Program.

Our intent in issuing this booklet is to illustrate how the chi-square test can contribute to GAO's work and to help auditors recognize when it can be used.

In GAO audits, two or more groups or subgroups are often compared for differences. For example, welfare recipients who receive job-related services might be compared to welfare recipients who received none to determine if the two groups differ in current employment status. Or members of armed forces reserve units who were satisfied with their drills might be compared with those who were dissatisfied to determine if satisfaction is related to grade level.

When the data from such comparisons are drawn from samples rather than from the entire population, some kind of statistical test should be used to determine the degree of confidence that can be attached to sample results. As used here, the expression"degree of confidence" is how sure we can be that the results would be similar if data were collected for the entire population instead of for a sample. The chi-square test for independence is one of the statistical tests to establish this. (See appendix I for the methods of computing and interpreting this test.)

This test is used when the available data is categorical, as opposed to continuous. Examples of categorical data are sex (male or female), race (often expressed in two or more categories), location (e.g., urban, suburban, or rural), and marital status (single, married, divorced, or separated). Continuous data, on the other hand, is data that can take any value on a scale (although the scale itself can have lower and upper limits), for example, weights and heights of adults, aptitude test scores, or baseball players' batting averages.

Although the chi-square test is used with categorical data, data that is basically continuous might be found recorded in categorical form only. For example, a program participant's income might only be recorded in categories, such as under $2,000, $2,000 to $5,000, or more than $5,000.

Even data that is recorded continuously may actually be more categorical than continuous; that is, it may cluster around certain points. For example, in an analysis of discharged veterans' years of military service, the scale could run from less than 1 year to over 40 years. However, examining the sample data may show that the years of service cluster around three points: 3 years (the period of service for one enlistment); 20 years (the point at which retirees get half pay); and 30 years (the point at which they can retire with full pay). When data clusters around a limited number of points on a scale, the appropriate test is the chi-square test, to be used after the data is arranged in categories.

CHAPTER 2

THE AUDIT APPROACH

The objective of the Law Enforcement Education Program (LEEP) is to improve the criminal justice system by providing educational opportunities to persons working in or considering criminal justice careers. The system includes the police force, probation and parole, the courts, and corrections. The program is administered by the Law Enforcement Assistance Administration, which provides money to colleges for loans and grants to students of criminal justice.

To help determine if LEEP's objective was being accomplished, GAO developed a questionnaire to obtain the opinions and experiences of former program participants. We obtained lists of students who had completed the program at 50 randomly selected schools in school years 1972 and 1973. From these lists, 550 persons were randomly selected to receive the questionnaire. We asked about participants' experience in the program, their present employment status, what they had learned, and how LEEP affected their decisions concerning a criminal justice career. Responses were received from 465 (85 percent) of the sample.

The chi-square test for independence was used several times in analyzing the responses. Two of these applications are detailed in chapter 3.

CHAPTER 3

APPLYING THE CHI-SQUARE TEST

The data showed that 96 respondents who had no criminal justice experience before entering the program sought such employment after completing the program. About 39 percent of these failed to find such employment. Table I breaks down the data by sex.

Table 1

Sex and Job-seeking Success of Participants (Number)

Sex	Found a job	Could not find a job	Total (note a)
Male	50	24	74
Female	7	13	20
	57	37	94

[a]Two respondents did not identify their sex and therefore are not included in the table.

The table shows that for our sample, sex is associated with success in finding a job in criminal justice--a larger proportion of males than females found jobs. This conclusion is more obvious when one looks at the percentages shown in table 2.

TABLE 2

Sex and Job-seeking Success of Participants (percent)

Sex	Found a job	Could not find a job	Total
Male	68	32	100
Female	35	65	100
	61	39	100

Tables 1 and 2 clearly show that females in our sample were much less likely to obtain a job in the criminal justice field than were males. But remember that we had only a

sample of participants. We had to find out whether this same result might be expected if the same data were obtained for all students who completed the program at the 50 colleges in 1972 and 1973. This is where the chi-square test was useful.

Chi-square compares the actual results with what probably would have happened if the two factors (in this case, sex and employment) were unrelated. The test requires computing a chi-square value and using a statistical table to interpret that value. We applied the chi-square test and found we could be 95-percent confident that our conclusion that sex is related to success in finding a job would hold true for the universe of participants.

Another aspect of the data concerned those employed in the criminal justice field at the time they answered the questionnaire. The data showed that most were working as police rather than in other parts of the criminal justice system. However, as shown in table 3, more of those with previous criminal justice experience were employed with the police than those without previous experience.

Table 3

Participants' Previous Criminal Justice Experience and Professional Area

Previous criminal justice experience	Professional area: Police	Other (note a)	Total	Percent becoming police
Yes	256	41	297	86
No	36	23	59	61
	292	64	356	82

[a]Includes probation and parole, courts, and corrections.

From a chi-square test, we found we could be 99-percent confident that the same conclusions would hold for the entire population.

The chi-square test results were the basis for statements included in our report to the Congress, "Problems in Administering Programs to Improve Law Enforcement Education" (GGD-75-67, June 11, 1975).

CHAPTER 4

OTHER APPLICATIONS

GAO has found the chi-square test to be a useful tool in evaluating social service programs such as the upward bound program. The technique also has been used in reviews of the aid to families with dependent children program, the developing institutions program within the Office of Education and others. In each case the test was used to establish and evaluate associations between the variables tested.

In reviewing the aid to families with dependent children program, GAO used the test to determine whether the services provided helped welfare recipients achieve self-support or reduced dependency. For example, in table 4 we can see that those welfare recipients who received developmental services did proportionately better, in terms of reduced dependency through employment, than those who did not. The chi-square test was used to establish that this association was not a product of chance related to our sample selection.

TABLE 4

Developmental Services
and Reduced Dependency Through Employment
(Denver, Colorado)

Services received?	Dependency reduced?		
	Yes	No	Total
Yes	14	62	76
No	0	49	49
	14	111	125

A somewhat different application of the chi-square test was involved in our review of the developing institutions program, which provides funds to "financially struggling" colleges and universities. GAO wanted to identify those factors influencing Office of Education choices of schools to participate in the program. The chi-square test was used to identify the variables that appeared to have some relationship to receiving funds, e.g., public vs. private or percentage of nonwhite enrollment. Variables identified as significantly related to whether or not the school received program funding, were then further analyzed to identify factors which tended to explain the amount of funds allocated. While the chi-square

test did not play a visible role in the second analysis, it did limit the number of variables considered.

GAO has used the chi-square test extensively to analyze data obtained from questionnaires. This use should not be surprising since most questionnaire data is in the form of categories or ranges. Typically, respondents indicate their answers by checking a block, thereby placing themselves in a category. The test is commonly used to see how answers to certain questions are associated. The review of the Law Enforcement Education Program described earlier is one example. Another example is a recent evaluation of State Employment Service operations in eight states. GAO tested the relationship between the type of job an applicant sought (white collar or blue collar) and whether the applicant contacted the Service when looking for a job.

This list of GAO's uses of the chi-square test for independence is by no means all inclusive. It is designed mainly to illustrate the variety of ways in which the test may be helpful. Generally speaking, whenever the data obtained from a sample is in the form of categories or ranges, or can be put into that form, and the analyst is interested in finding a relationship between two of the variables, he should consider using the chi-square test for independence.

COMPUTATION AND INTERPRETATION OF CHI-SQUARE VALUES

The chi-square test for independence involves computing a chi-square value and then using a statistical table to interpret that value. Computing the chi-square value requires four steps. These steps will be illustrated by using the data from table 3, which is as follows:

Previous criminal justice experience	Professional area		
	Police	Other	Total
Yes	256	41	297
No	36	23	59
	292	64	356

For the purpose of the chi-square computation, this table has two rows and two columns (totals not included) and four cells. The "Police" column has two cells ("Yes" and "No") and the "Other" column has two cells. The steps in the computation are as follows:

1. For each of the cells, compute the expected value--the value that would be found if one variable had no relation to or association with the other variable. (The present entries in the table--256, 36, 41, and 23--are known as the observed values.) The expected value for a cell is computed by multiplying

$$\frac{\text{column total}}{\text{total sample}} \times \frac{\text{row total}}{\text{total sample}} \times \text{Total sample}$$

The expected value for each of the four cells are computed as follows:

Cell							Expected Value
Column 1, row 1	$\frac{292}{356}$	X	$\frac{297}{356}$	X	356	=	243.6
Column 1, row 2	$\frac{292}{356}$	X	$\frac{59}{356}$	X	356	=	48.4
Column 2, row 1	$\frac{64}{356}$	X	$\frac{297}{356}$	X	356	=	53.4
Column 2, row 2	$\frac{64}{356}$	X	$\frac{59}{356}$	X	356	=	10.6

If the expected values are put in a table and expressed also in percentages, the table would be as follows:

Previous criminal justice experience	Police		Other		Total
	Number	Percent	Number	Percent	
Yes	243.6	82	53.4	18	297.0
No	48.4	82	10.6	18	59.0
	292.0	82	64.0	18	356.0

2. For each cell, subtract the expected value from the observed value. Then, since this table has four cells, subtract 0.5 from the difference between the observed and expected value to obtain the corrected difference. (If the table has six or more cells, the correction factor is not used. The reason for using a correction factor in a four-cell table will not be explained in this case study.) Square the corrected differences. The following table shows the difference, the corrected difference, and the corrected difference squared for the data in table 3.

Cell	Observed	Expected	Difference (note a)	Corrected Difference	Corrected difference squared
Column 1, row 1	256	243.6	12.4	11.9	141.6
Column 1, row 2	36	48.4	12.4	11.9	141.6
Column 2, row 1	41	53.4	12.4	11.9	141.6
Column 2, row 2	23	10.6	12.4	11.9	141.6

[a]In the chi-square computation, all differences are assumed to be positive

3. Divide the difference squared for each cell by the expected value. For table 3, these quotients are:

Column 1, row 1	0.581
Column 1, row 2	2.926
Column 2, row 1	2.652
Column 2, row 2	13.358

4. Add the quotients for each of the cells. The sum is the chi-square value. In our example, it is 19.517.

The chi-square value is essentially a measure of the degree to which the observed cell values differ in total from the values that would have been found had the variables not been correlated.

To interpret the chi-square value, a table showing chi-square distribution is needed. The higher the chi-square value, the higher the likelihood of a relationship. The following is a portion of this table:

Degrees of freedom	Confidence Level .90	.95	.99
1	2.706	3.841	6.635
2	4.605	5.991	9.210
3	6.251	7.815	11.345
4	7.779	9.488	13.277

The first step in interpreting the chi-square value is to compute degrees of freedom by multiplying the number of columns in the table minus one by the number of rows minus one. Table 3 has two columns and two rows. Therefore, the number of degrees of freedom is 1. We then look at the row in the distribution table for 1 degree of freedom and find that our chi-square value of 19.517 far exceeds the value that would indicate a 99-percent confidence level. Therefore, we conclude that the difference between those with previous criminal justice experience and those with no such experience is significant at more than a 99-percent confidence level. We are thus very sure that the difference would hold for the entire population and not only for our sample.

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